

Algebraic Number Theory

(PARI-GP version 2.13.1)

Binary Quadratic Forms

create $ax^2 + bxy + cy^2$ (distance d) **qfb**($a, b, c, \{d\}$)
reduce x ($s = \sqrt{D}$, $l = \lfloor s \rfloor$) **qfbred**($x, \{flag\}, \{D\}, \{l\}, \{s\}$)
return $[y, g]$, $g \in \text{SL}_2(\mathbf{Z})$, $y = g \cdot x$ reduced **qfbreds12**(x)
composition of forms $x*y$ or **qfbnucomp**(x, y, l)
 n -th power of form x^n or **qfbnupow**(x, n)
composition without reduction **qfbcomprow**(x, y)
 n -th power without reduction **qfbpowrow**(x, n)
prime form of disc. x above prime p **qfbprimeform**(x, p)
class number of disc. x **qfbclassno**(x)
Hurwitz class number of disc. x **qfbhclassno**(x)
solve $Q(x, y) = n$ in integers **qfbsolve**(Q, n)

Quadratic Fields

quadratic number $\omega = \sqrt{x}$ or $(1 + \sqrt{x})/2$ **quadgen**(x)
minimal polynomial of ω **quadpoly**(x)
discriminant of $\mathbf{Q}(\sqrt{x})$ **quaddisc**(x)
regulator of real quadratic field **quadregulator**(x)
fundamental unit in real $\mathbf{Q}(\sqrt{D})$ **quadunit**($D, \{w\}$)
class group of $\mathbf{Q}(\sqrt{D})$ **quadclassunit**($D, \{flag\}, \{t\}$)
Hilbert class field of $\mathbf{Q}(\sqrt{D})$ **quadhilbert**($D, \{flag\}$)
... using specific class invariant ($D < 0$) **polclass**($D, \{inv\}$)
ray class field modulo f of $\mathbf{Q}(\sqrt{D})$ **quadrays**($D, f, \{flag\}$)

General Number Fields: Initializations

The number field $K = \mathbf{Q}[X]/(f)$ is given by irreducible $f \in \mathbf{Q}[X]$. We denote $\theta = \bar{X}$ the canonical root of f in K . A nf structure contains a maximal order and allows operations on elements and ideals. A bnf adds class group and units. A bnr is attached to ray class groups and class field theory. A rmf is attached to relative extensions L/K .

init number field structure nf **nfinit**($f, \{flag\}$)
known integer basis B **nfinit**($[f, B]$)
order maximal at $vp = [p_1, \dots, p_k]$ **nfinit**($[f, vp]$)
order maximal at all $p \leq P$ **nfinit**($[f, P]$)
certify maximal order **nfcertify**(nf)

nf members:

a monic $F \in \mathbf{Z}[X]$ defining K $nf.pol$
number of real/complex places $nf.r1/r2/sign$
discriminant of nf $nf.disc$
primes ramified in nf $nf.p$
 T_2 matrix $nf.t2$
complex roots of F $nf.roots$
integral basis of \mathbf{Z}_K as powers of θ $nf.zk$
different/codifferent $nf.diff, nf.codiff$
index $[\mathbf{Z}_K : \mathbf{Z}[X]/(F)]$ $nf.index$
recompute nf using current precision **nfnewprec**(nf)
init relative rmf $L = K[Y]/(g)$ **rnfin**(nf, g)
init bnf structure **bnfinit**(f, l)

bnf members:

same as nf , plus
underlying nf $bnf.nf$
class group, regulator $bnf.clgp, bnf.reg$
fundamental/torsion units $bnf.fu, bnf.tu$
add S -class group and units, yield $bnfS$ **bnfsunit**(bnf, S)

init class field structure bnr **bnrinit**($bnf, m, \{flag\}$)
bnr members: same as bnf , plus
underlying bnf $bnr.bnf$
big ideal structure $bnr.bid$
modulus m $bnr.mod$
structure of $(\mathbf{Z}_K/m)^*$ $bnr.zkst$

Fields, subfields, embeddings

Defining polynomials, embeddings
smallest poly defining $f = 0$ (slow) **polredabs**($f, \{flag\}$)
small poly defining $f = 0$ (fast) **polredbest**($f, \{flag\}$)
random Tschirnhausen transform of f **poltschirnhaus**(f)
 $\mathbf{Q}[t]/(f) \subset \mathbf{Q}[t]/(g)$? Isomorphic? **nfisincl**(f, g), **nfisisom**
reverse polmod $a = A(t) \bmod T(t)$ **modreverse**(a)
compositum of $\mathbf{Q}[t]/(f)$, $\mathbf{Q}[t]/(g)$ **polcompositum**($f, g, \{flag\}$)
compositum of $K[t]/(f)$, $K[t]/(g)$ **nfcompositum**($nf, f, g, \{flag\}$)
splitting field of K (degree divides d) **nfsplitting**($nf, \{d\}$)
signs of real embeddings of x **nfeltsign**($nf, x, \{pl\}$)
complex embeddings of x **nfeltembed**($nf, x, \{pl\}$)
 $T \in K[t]$, # of real roots of $\sigma(T) \in R[t]$ **nfpolsturm**($nf, T, \{pl\}$)

Subfields, polynomial factorization

subfields (of degree d) of nf **nfsubfields**($nf, \{d\}$)
maximal subfields of nf **nfsubfieldsmax**(nf)
maximal CM subfield of nf **nfsubfieldscm**(nf)
 d -th degree subfield of $\mathbf{Q}(\zeta_n)$ **polsubcyclo**($n, d, \{v\}$)
roots of unity in nf **nfroots of 1**(nf)
roots of g belonging to nf **nfroots**(nf, g)
factor g in nf **nffactor**(nf, g)

Linear and algebraic relations

poly of degree $\leq k$ with root $x \in \mathbf{C}$ **algdep**(x, k)
alg. dep. with pol. coeffs for series s **seralgdep**(s, x, y)
small linear rel. on coords of vector x **lindep**(x)

Basic Number Field Arithmetic (nf)

Number field elements are **t_INT**, **t_FRAC**, **t_POL**, **t_POLMOD**, or **t_COL** (on integral basis $nf.zk$).

Basic operations

$x + y$ **nfeltadd**(nf, x, y)
 $x \times y$ **nfeltmul**(nf, x, y)
 x^n , $n \in \mathbf{Z}$ **nfeltpow**(nf, x, n)
 x/y **nfeltdiv**(nf, x, y)
 $q = x \setminus y := \text{round}(x/y)$ **nfeltdivu**(nf, x, y)
 $r = x \% y := x - (x \setminus y)y$ **nfeltmod**(nf, x, y)
... $[q, r]$ as above **nfeltdivrem**(nf, x, y)
reduce x modulo ideal A **nfeltreduce**(nf, x, A)
absolute trace $\text{Tr}_{K/\mathbf{Q}}(x)$ **nfelttrace**(nf, x)
absolute norm $N_{K/\mathbf{Q}}(x)$ **nfeltnorm**(nf, x)

Multiplicative structure of K^* ; $K^*/(K^*)^n$

valuation $v_p(x)$ **nfeltval**(nf, x, p)
... write $x = \pi^{v_p(x)}y$ **nfeltval**($nf, x, p, \&y$)
quadratic Hilbert symbol (at p) **nfhilbert**($nf, a, b, \{p\}$)
 b such that $xb^n = v$ is small **idealredmodpower**(nf, x, n)

Maximal order and discriminant

integral basis of field $\mathbf{Q}[x]/(f)$ **nfbasis**(f)
field discriminant of $\mathbf{Q}[x]/(f)$ **nfdisc**(f)
... and factorization **nfdiscfactors**(f)
express x on integer basis **nfalgtobasis**(nf, x)
express element x as a polmod **nfbasistoalg**(nf, x)

Dedekind Zeta Function ζ_K , Hecke L series

$R = [c, w, h]$ in initialization means we restrict $s \in \mathbf{C}$ to domain $|\Re(s) - c| < w$, $|\Im(s)| < h$; $R = [w, h]$ encodes $[1/2, w, h]$ and $[h]$ encodes $R = [1/2, 0, h]$ (critical line up to height h).

ζ_K as Dirichlet series, $N(I) < b$ **dirzetak**(nf, b)

init $\zeta_K^{(k)}(s)$ for $k \leq n$ **L = lfunit**($bnf, R, \{n = 0\}$)
compute $\zeta_K(s)$ (n -th derivative) **lfun**($L, s, \{n = 0\}$)
compute $\Lambda_K(s)$ (n -th derivative) **lfunlambda**($L, s, \{n = 0\}$)

init $L_K^{(k)}(s, \chi)$ for $k \leq n$ **L = lfunit**($[bnr, chi], R, \{n = 0\}$)
compute $L_K(s, \chi)$ (n -th derivative) **lfun**($L, s, \{n\}$)
Artin root number of K **bnrrootnumber**($bnr, chi, \{flag\}$)
 $L(1, \chi)$, for all χ trivial on H **bnrL1**($bnr, \{H\}, \{flag\}$)

Class Groups & Units (bnf, bnr)

Class field theory data $a_1, \{a_2\}$ is usually bnr (ray class field), bnr, H (congruence subgroup) or bnr, χ (character on **bnr.clgp**). Any of these define a unique abelian extension of K .

units / S -units **bnfunits**($bnf, \{S\}$)
remove GRH assumption from bnf **bnfcertify**(bnf)
expo. of ideal x on class gp **bnfisprincipal**($bnf, x, \{flag\}$)
expo. of ideal x on ray class gp **bnrisprincipal**($bnr, x, \{flag\}$)
expo. of x on fund. units **bnfisunit**(bnf, x)
... on S -units, U is **bnfunits**(bnf, S) **bnfisunit**($bnfs, U$)
signs of real embeddings of $bnf.fu$ **bnfsignunit**(bnf)
narrow class group **bnfnarrow**(bnf)

Class Field Theory

ray class number for modulus m **bnrclassno**(bnf, m)
discriminant of class field **bnrdisc**($a_1, \{a_2\}$)
ray class numbers, l list of moduli **bnrclassnolist**(bnf, l)
discriminants of class fields **bnrdisc**($bnf, l, \{arch\}, \{flag\}$)
decode output from **bnrdisc**list **bnfdecodemodule**(nf, fa)
is modulus the conductor? **bnrisconductor**($a_1, \{a_2\}$)
is class field (bnr, H) Galois over K^G **bnrisgalois**(bnr, G, H)
action of automorphism on **bnr.gen** **bnrgaloismatrix**(bnr, aut)
apply **bnrgaloismatrix** M to H **bnrgaloisapply**(bnr, M, H)
characters on **bnr.clgp** s.t. $\chi(g_i) = e(v_i)$ **bnrchar**($bnr, g, \{v\}$)
conductor of character χ **bnrconductor**(bnr, chi)
conductor of extension **bnrconductor**($a_1, \{a_2\}, \{flag\}$)
conductor of extension $K[Y]/(g)$ **rnfconductor**(bnf, g)
canonical projection $\text{Cl}_F \rightarrow \text{Cl}_f$, $f \mid F$ **bnrmmap**
Artin group of extension $K[Y]/(g)$ **rnfnormgroup**(bnr, g)
subgroups of bnr , index $\leq b$ **subgroup**($bnr, b, \{flag\}$)
class field defined by $H \subset \text{Cl}_f$ **bnrclassfield**(bnr, H)
... low level equivalent, prime degree **rnfkummer**(bnr, H)
same, using Stark units (real field) **bnrstark**($bnr, sub, \{flag\}$)
is a an n -th power in K_v ? **nfislocalpower**(nf, v, a, n)
cyclic L/K satisf. local conditions **nfgrunwaldwang**(nf, P, D, pl)

Logarithmic class group

logarithmic ℓ -class group	<code>bnflog(<i>bnf</i>, ℓ)</code>
$[\tilde{e}(F_v/Q_p), \tilde{f}(F_v/Q_p)]$	<code>bnflog_{ef}(<i>bnf</i>, <i>pr</i>)</code>
$\exp \deg_F(A)$	<code>bnflogdegree(<i>bnf</i>, <i>A</i>, ℓ)</code>
is ℓ -extension L/K locally cyclotomic	<code>rnfislocalcyclo(<i>rnf</i>)</code>

Ideals: elements, primes, or matrix of generators in HNF

is id an ideal in nf ?	<code>nfisideal(<i>nf</i>, <i>id</i>)</code>
is x principal in bnf ?	<code>bnfisprincipal(<i>bnf</i>, <i>x</i>)</code>
give $[a, b]$, s.t. $a\mathbf{Z}_K + b\mathbf{Z}_K = x$	<code>idealtwoelt(<i>nf</i>, <i>x</i>, {<i>a</i>})</code>
put ideal a ($a\mathbf{Z}_K + b\mathbf{Z}_K$) in HNF form	<code>idealhnf(<i>nf</i>, <i>a</i>, {<i>b</i>})</code>
norm of ideal x	<code>idealn_{orm}(<i>nf</i>, <i>x</i>)</code>
minimum of ideal x (direction v)	<code>idealmin(<i>nf</i>, <i>x</i>, <i>v</i>)</code>
LLL-reduce the ideal x (direction v)	<code>idealred(<i>nf</i>, <i>x</i>, {<i>v</i>})</code>

Ideal Operations

add ideals x and y	<code>idealadd(<i>nf</i>, <i>x</i>, <i>y</i>)</code>
multiply ideals x and y	<code>idealmul(<i>nf</i>, <i>x</i>, <i>y</i>, {<i>flag</i>})</code>
intersection of ideal x with Q	<code>idealdown(<i>nf</i>, <i>x</i>)</code>
intersection of ideals x and y	<code>idealintersect(<i>nf</i>, <i>x</i>, <i>y</i>, {<i>flag</i>})</code>
n -th power of ideal x	<code>idealpow(<i>nf</i>, <i>x</i>, <i>n</i>, {<i>flag</i>})</code>
inverse of ideal x	<code>idealin_v(<i>nf</i>, <i>x</i>)</code>
divide ideal x by y	<code>idealdiv(<i>nf</i>, <i>x</i>, <i>y</i>, {<i>flag</i>})</code>
Find $(a, b) \in x \times y$, $a + b = 1$	<code>idealaddtoone(<i>nf</i>, <i>x</i>, {<i>y</i>})</code>
coprime integral A, B such that $x = A/B$	<code>idealnumden(<i>nf</i>, <i>x</i>)</code>

Primes and Multiplicative Structure

check whether x is a maximal ideal	<code>idealismaximal(<i>nf</i>, <i>x</i>)</code>
factor ideal x in \mathbf{Z}_K	<code>idealfactor(<i>nf</i>, <i>x</i>)</code>
expand ideal factorization in K	<code>idealfactorback(<i>nf</i>, <i>f</i>, {<i>e</i>})</code>
is ideal A an n -th power ?	<code>idealispower(<i>nf</i>, <i>A</i>, <i>n</i>)</code>
expand elt factorization in K	<code>nffactorback(<i>nf</i>, <i>f</i>, {<i>e</i>})</code>
decomposition of prime p in \mathbf{Z}_K	<code>idealprimedec(<i>nf</i>, <i>p</i>)</code>
valuation of x at prime ideal pr	<code>idealval(<i>nf</i>, <i>x</i>, <i>pr</i>)</code>
weak approximation theorem in nf	<code>idealchinese(<i>nf</i>, <i>x</i>, <i>y</i>)</code>
$a \in K$, s.t. $v_{\mathfrak{p}}(a) = v_{\mathfrak{p}}(x)$ if $v_{\mathfrak{p}}(x) \neq 0$	<code>idealappr(<i>nf</i>, <i>x</i>)</code>
$a \in K$ such that $(a \cdot x, y) = 1$	<code>idealcoprime(<i>nf</i>, <i>x</i>, <i>y</i>)</code>
give bid =structure of $(\mathbf{Z}_K/id)^*$	<code>idealstar(<i>nf</i>, <i>id</i>, {<i>flag</i>})</code>
structure of $(1 + \mathfrak{p})/(1 + \mathfrak{p}^k)$	<code>idealprincipalunits(<i>nf</i>, <i>pr</i>, <i>k</i>)</code>
discrete log of x in $(\mathbf{Z}_K/bid)^*$	<code>ideallog(<i>nf</i>, <i>x</i>, <i>bid</i>)</code>
idealstar of all ideals of norm $\leq b$	<code>ideallist(<i>nf</i>, <i>b</i>, {<i>flag</i>})</code>
add Archimedean places	<code>ideallistarch(<i>nf</i>, <i>b</i>, {<i>ar</i>}, {<i>flag</i>})</code>
init modpr structure	<code>nfmodprinit(<i>nf</i>, <i>pr</i>, {<i>v</i>})</code>
project t to \mathbf{Z}_K/pr	<code>nfmodpr(<i>nf</i>, <i>t</i>, <i>modpr</i>)</code>
lift from \mathbf{Z}_K/pr	<code>nfmodprlift(<i>nf</i>, <i>t</i>, <i>modpr</i>)</code>

Galois theory over Q

conjugates of a root θ of nf	<code>nfgaloisconj(<i>nf</i>, {<i>flag</i>})</code>
apply Galois automorphism s to x	<code>nfgaloisapply(<i>nf</i>, <i>s</i>, <i>x</i>)</code>
Galois group of field $\mathbf{Q}[x]/(f)$	<code>polgalois(<i>f</i>)</code>
initializes a Galois group structure G	<code>galoisinit(<i>pol</i>, {<i>den</i>})</code>
character table of G	<code>galoischartable(<i>G</i>)</code>
conjugacy classes of G	<code>galoisconjugacyclasses(<i>G</i>)</code>
$\det(1 - \rho(g)T)$, χ character of ρ	<code>galoischarpoly(<i>G</i>, χ, {<i>o</i>})</code>
$\det(\rho(g))$, χ character of ρ	<code>galoischar_{det}(<i>G</i>, χ, {<i>o</i>})</code>
action of p in nfgaloisconj form	<code>galoisperm_{topol}(<i>G</i>, {<i>p</i>})</code>
identify as abstract group	<code>galoisidentify(<i>G</i>)</code>
export a group for GAP/MAGMA	<code>galoisexport(<i>G</i>, {<i>flag</i>})</code>
subgroups of the Galois group G	<code>galoissubgroups(<i>G</i>)</code>
is subgroup H normal?	<code>galoisisnormal(<i>G</i>, <i>H</i>)</code>

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subfields from subgroups	<code>galoissubfields(<i>G</i>, {<i>flag</i>}, {<i>v</i>})</code>
fixed field	<code>galoisfixedfield(<i>G</i>, <i>perm</i>, {<i>flag</i>}, {<i>v</i>})</code>
Frobenius at maximal ideal P	<code>idealfrobenius(<i>nf</i>, <i>G</i>, <i>P</i>)</code>
ramification groups at P	<code>idealramgroups(<i>nf</i>, <i>G</i>, <i>P</i>)</code>
is G abelian?	<code>galoisisabelian(<i>G</i>, {<i>flag</i>})</code>
abelian number fields/ \mathbf{Q}	<code>galoissubcyclo(<i>N</i>, <i>H</i>, {<i>flag</i>}, {<i>v</i>})</code>

The galpol package

query the package: polynomial	<code>galoisgetpol(<i>a</i>, <i>b</i>, {<i>s</i>})</code>
... : permutation group	<code>galoisgetgroup(<i>a</i>, <i>b</i>)</code>
... : group description	<code>galoisgetname(<i>a</i>, <i>b</i>)</code>

Relative Number Fields (rnf)

Extension L/K is defined by $T \in K[x]$.

absolute equation of L	<code>rnfequation(<i>nf</i>, <i>T</i>, {<i>flag</i>})</code>
is L/K abelian?	<code>rnfisabelian(<i>nf</i>, <i>T</i>)</code>
relative nfalgtobasis	<code>rnfalgtobasis(<i>rnf</i>, <i>x</i>)</code>
relative nfbasistoalg	<code>rnfbasistoalg(<i>rnf</i>, <i>x</i>)</code>
relative idealhnf	<code>rnfidealhnf(<i>rnf</i>, <i>x</i>)</code>
relative idealmul	<code>rnfidealmul(<i>rnf</i>, <i>x</i>, <i>y</i>)</code>
relative idealtwoelt	<code>rnfidealtwoelt(<i>rnf</i>, <i>x</i>)</code>

Lifts and Push-downs

absolute \rightarrow relative representation for x	<code>rnfeltabstorel(<i>rnf</i>, <i>x</i>)</code>
relative \rightarrow absolute representation for x	<code>rnfeltreltoabs(<i>rnf</i>, <i>x</i>)</code>
lift x to the relative field	<code>rnfeltup(<i>rnf</i>, <i>x</i>)</code>
push x down to the base field	<code>rnfeltdown(<i>rnf</i>, <i>x</i>)</code>
idem for x ideal: (rnfideal)reltoabs, abstorel, up, down	

Norms and Trace

relative norm of element $x \in L$	<code>rnfeltnorm(<i>rnf</i>, <i>x</i>)</code>
relative trace of element $x \in L$	<code>rnfelttrace(<i>rnf</i>, <i>x</i>)</code>
absolute norm of ideal x	<code>rnfidealn_{orm}abs(<i>rnf</i>, <i>x</i>)</code>
relative norm of ideal x	<code>rnfidealn_{orm}rel(<i>rnf</i>, <i>x</i>)</code>
solutions of $N_{K/\mathbf{Q}}(y) = x \in \mathbf{Z}$	<code>bnfisintnorm(<i>bnf</i>, <i>x</i>)</code>
is $x \in \mathbf{Q}$ a norm from K ?	<code>bnfisnorm(<i>bnf</i>, <i>x</i>, {<i>flag</i>})</code>
initialize T for norm eq. solver	<code>rnfisnorminit(<i>K</i>, <i>pol</i>, {<i>flag</i>})</code>
is $a \in K$ a norm from L ?	<code>rnfisnorm(<i>T</i>, <i>a</i>, {<i>flag</i>})</code>
initialize t for Thue equation solver	<code>thueinit(<i>f</i>)</code>
solve Thue equation $f(x, y) = a$	<code>thue(<i>t</i>, <i>a</i>, {<i>sol</i>})</code>
characteristic poly. of a mod T	<code>rnfcharpoly(<i>nf</i>, <i>T</i>, <i>a</i>, {<i>v</i>})</code>

Factorization

factor ideal x in L	<code>rnfidealfactor(<i>rnf</i>, <i>x</i>)</code>
$[S, T]: T_{i,j} \mid S_i$; S primes of K above p	<code>rnfidealprimedec(<i>rnf</i>, <i>p</i>)</code>

Maximal order \mathbf{Z}_L as a \mathbf{Z}_K -module

relative polredbest	<code>rnfpolredbest(<i>nf</i>, <i>T</i>)</code>
relative polredabs	<code>rnfpolredabs(<i>nf</i>, <i>T</i>)</code>
relative Dedekind criterion, prime pr	<code>rnfdedekind(<i>nf</i>, <i>T</i>, <i>pr</i>)</code>
discriminant of relative extension	<code>rnfdisc(<i>nf</i>, <i>T</i>)</code>
pseudo-basis of \mathbf{Z}_L	<code>rnfpseudobasis(<i>nf</i>, <i>T</i>)</code>

General \mathbf{Z}_K -modules: $M = [\text{matrix, vec. of ideals}] \subset L$

relative HNF / SNF	<code>nfhnf(<i>nf</i>, <i>M</i>), nfsnf</code>
multiple of det M	<code>nf_{det}tint(<i>nf</i>, <i>M</i>)</code>
HNF of M where $d = nfdettint(M)$	<code>nfhnfmod(<i>x</i>, <i>d</i>)</code>
reduced basis for M	<code>rnfilllgram(<i>nf</i>, <i>T</i>, <i>M</i>)</code>
determinant of pseudo-matrix M	<code>rnfdet(<i>nf</i>, <i>M</i>)</code>
Steinitz class of M	<code>rnfst_{ein}itz(<i>nf</i>, <i>M</i>)</code>

\mathbf{Z}_K -basis of M if \mathbf{Z}_K -free, or 0

n -basis of M , or $(n + 1)$ -generating set

is M a free \mathbf{Z}_K -module?

`rnfhnbasis(bnf, M)`

`rnfbasis(bnf, M)`

`rnfisfree(bnf, M)`

Associative Algebras

A is a general associative algebra given by a multiplication table mt (over \mathbf{Q} or \mathbf{F}_p); represented by al from `algttableinit`.

create al from mt (over \mathbf{F}_p)	<code>algt_{table}init(<i>mt</i>, {<i>p</i> = 0})</code>
group algebra $\mathbf{Q}[G]$ (or $\mathbf{F}_p[G]$)	<code>algg_{roup}(<i>G</i>, {<i>p</i> = 0})</code>
center of group algebra	<code>algg_{roup}center(<i>G</i>, {<i>p</i> = 0})</code>

Properties

is (mt, p) OK for <code>algt_{table}init</code> ?	<code>algisassociative(<i>mt</i>, {<i>p</i> = 0})</code>
multiplication table mt	<code>algm_{ult}table(<i>al</i>)</code>
dimension of A over prime subfield	<code>algdim(<i>al</i>)</code>
characteristic of A	<code>algchar(<i>al</i>)</code>
is A commutative?	<code>algiscommutative(<i>al</i>)</code>
is A simple?	<code>algissimple(<i>al</i>)</code>
is A semi-simple?	<code>algissemisimple(<i>al</i>)</code>
center of A	<code>algcenter(<i>al</i>)</code>
Jacobson radical of A	<code>algradical(<i>al</i>)</code>
radical J and simple factors of A/J	<code>algsimpledec(<i>al</i>)</code>

Operations on algebras

create A/I , I two-sided ideal	<code>algquotient(<i>al</i>, <i>I</i>)</code>
create $A_1 \otimes A_2$	<code>algtensor(<i>al</i>₁, <i>al</i>₂)</code>
create subalgebra from basis B	<code>algsubalg(<i>al</i>, <i>B</i>)</code>
quotients by ortho. central idempotents e	<code>algcentralproj(<i>al</i>, <i>e</i>)</code>
isomorphic alg. with integral mult. table	<code>algmakeintegral(<i>mt</i>)</code>
prime subalgebra of semi-simple A over \mathbf{F}_p	<code>algprimesubalg(<i>al</i>)</code>
find isomorphism $A \cong M_d(\mathbf{F}_q)$	<code>algsplit(<i>al</i>)</code>

Operations on lattices in algebras

lattice generated by cols. of M	<code>alglathnf(<i>al</i>, <i>M</i>)</code>
... by the products xy , $x \in lat_1$, $y \in lat_2$	<code>alglatmul(<i>al</i>, <i>lat</i>₁, <i>lat</i>₂)</code>
sum $lat_1 + lat_2$ of the lattices	<code>alglatadd(<i>al</i>, <i>lat</i>₁, <i>lat</i>₂)</code>
intersection $lat_1 \cap lat_2$	<code>alglatinter(<i>al</i>, <i>lat</i>₁, <i>lat</i>₂)</code>
test $lat_1 \subset lat_2$	<code>alglatsubset(<i>al</i>, <i>lat</i>₁, <i>lat</i>₂)</code>
generalized index $(lat_2 : lat_1)$	<code>alglatindex(<i>al</i>, <i>lat</i>₁, <i>lat</i>₂)</code>
$\{x \in al \mid x \cdot lat_1 \subset lat_2\}$	<code>alglatlefttransporter(<i>al</i>, <i>lat</i>₁, <i>lat</i>₂)</code>
$\{x \in al \mid lat_1 \cdot x \subset lat_2\}$	<code>alglatrightrighttransporter(<i>al</i>, <i>lat</i>₁, <i>lat</i>₂)</code>
test $x \in lat$ (set c = coord. of x)	<code>alglatcontains(<i>al</i>, <i>lat</i>, <i>x</i>, {&<i>c</i>})</code>
element of lat with coordinates c	<code>alglatelement(<i>al</i>, <i>lat</i>, <i>c</i>)</code>

Operations on elements

$a + b$, $a - b$, $-a$	<code>algadd(<i>al</i>, <i>a</i>, <i>b</i>)</code> , <code>algsub</code> , <code>algneg</code>
$a \times b$, a^2	<code>algmul(<i>al</i>, <i>a</i>, <i>b</i>)</code> , <code>algsqr</code>
a^n , a^{-1}	<code>algpow(<i>al</i>, <i>a</i>, <i>n</i>)</code> , <code>algin_v</code>
is x invertible ? (then set $z = x^{-1}$)	<code>algisinv(<i>al</i>, <i>x</i>, {&<i>z</i>})</code>
find z such that $x \times z = y$	<code>algdivl(<i>al</i>, <i>x</i>, <i>y</i>)</code>
find z such that $z \times x = y$	<code>algdivr(<i>al</i>, <i>x</i>, <i>y</i>)</code>
does x s.t. $x \times z = y$ exist? (set it)	<code>algisdivl(<i>al</i>, <i>x</i>, <i>y</i>, {&<i>z</i>})</code>
matrix of $v \mapsto x \cdot v$	<code>algtomatrix(<i>al</i>, <i>x</i>)</code>
absolute norm	<code>algnorm(<i>al</i>, <i>x</i>)</code>
absolute trace	<code>algtrace(<i>al</i>, <i>x</i>)</code>
absolute char. polynomial	<code>algcharpoly(<i>al</i>, <i>x</i>)</code>
given $a \in A$ and polynomial T , return $T(a)$	<code>algpoleval(<i>al</i>, <i>T</i>, <i>a</i>)</code>
random element in a box	<code>algrandom(<i>al</i>, <i>b</i>)</code>

Based on an earlier version by Joseph H. Silverman

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Send comments and corrections to (Karim.Belabas@math.u-bordeaux.fr)

Central Simple Algebras

A is a central simple algebra over a number field K ; represented by al from **alginit**; K is given by a nf structure.
create CSA from data **alginit**($B, C, \{v\}, \{maxord = 1\}$)
multiplication table over K $B = K, C = mt$
cyclic algebra ($L/K, \sigma, b$) $B = rnf, C = [sigma, b]$
quaternion algebra $(a, b)_K$ $B = K, C = [a, b]$
matrix algebra $M_d(K)$ $B = K, C = d$
local Hasse invariants over K $B = K, C = [d, [PR, HF], HI]$

Properties

type of al (mt, CSA) **algtype**(al)
dimension of A over \mathbf{Q} **algdim**($al, 1$)
dimension of al over its center K **algdim**(al)
degree of A ($= \sqrt{\dim_K A}$) **algdegree**(al)
 al a cyclic algebra ($L/K, \sigma, b$); return σ **algaut**(al)
...return b **algb**(al)
...return L/K , as an rnf **algsplittingfield**(al)
split A over an extension of K **algsplittingdata**(al)
splitting field of A as an rnf over center **algsplittingfield**(al)
multiplication table over center **algrelmultable**(al)
places of K at which A ramifies **algramifiedplaces**(al)
Hasse invariants at finite places of K **alghassef**(al)
Hasse invariants at infinite places of K **alghassei**(al)
Hasse invariant at place v **alghasse**(al, v)
index of A over K (at place v) **algindex**($al, \{v\}$)
is al a division algebra? (at place v) **algisdivision**($al, \{v\}$)
is A ramified? (at place v) **algisramified**($al, \{v\}$)
is A split? (at place v) **algissplit**($al, \{v\}$)

Operations on elements

reduced norm **algnorm**(al, x)
reduced trace **algtrace**(al, x)
reduced char. polynomial **algcharpoly**(al, x)
express x on integral basis **algalgtobasis**(al, x)
convert x to algebraic form **algbasistoalg**(al, x)
map $x \in A$ to $M_d(L)$, L split. field **algtomatrix**(al, x)

Orders

Z-basis of order \mathcal{O}_0 **algbasis**(al)
discriminant of order \mathcal{O}_0 **algdisc**(al)
Z-basis of natural order in terms \mathcal{O}_0 's basis **alginvbasis**(al)